Plan of Lectures

I. **Overview**
   - Lecture duration $\sim$ 1 hr

II. **What are GWs?**
   - Lecture duration $\sim$ 2 hr

III. **Gravity Tests with GWs**
   - Lecture duration $\sim$ 1.5 hr

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That would be one of the most fascinating things man could do, because it would tell you very much how the universe started.

— Rainer Weiss

北京大学
PEKING UNIVERSITY

Frontiers of GWs (III): Testing GR

Kavli Institute for Astronomy and Astrophysics

Lijing Shao (邵立晶)

KITS Summer School · 江苏溧阳

References

- **Matched Filter**
  - Finn, PRD 46 (1992) 5236
  - Cutler & Flanagan, PRD 49 (1994) 2658
  - Cutler, PRD 57 (1998) 7089
- **LIGO/Virgo Collaboration**
  - **GW150914 & GW170817**: PRL 116:221101; PRL 123:011102
  - **GWTC-1 & GWTC-2**: PRD 100:104036; PRD 103:122002
- **Review Papers**[1]
  - Berti *et al.*, CQG 32 (2015) 243001
  - Yunes, Yagi, & Pretorius, PRD 94 (2016) 8

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[1] Both are quite long.

Modern Physics Landscape

- **Standard Model**

quantum field theory

- **General Relativity**

gravitation and spacetime

How the Universe is Ruled

- **Particles of strong, weak, electromagnetic interactions**

$$\begin{aligned}
\mathcal{L}_{\text{lepton}} &= \frac{1}{2} i e e_a^\mu \left[\bar{L}_A \gamma^a \overleftrightarrow{D}_\mu L_A + \bar{R}_A \gamma^a \overleftrightarrow{D}_\mu R_A\right] \\
\mathcal{L}_{\text{quark}} &= \frac{1}{2} i e e_a^\mu \left[\bar{Q}_A \gamma^a \overleftrightarrow{D}_\mu Q_A + \bar{U}_A \gamma^a \overleftrightarrow{D}_\mu U_A + \bar{D}_A \gamma^a \overleftrightarrow{D}_\mu D_A\right] \\
\mathcal{L}_{\text{Yukawa}} &= -e\left[(G_L)_{AB}\, \bar{L}_A \varphi R_B + (G_U)_{AB}\, \bar{Q}_A \varphi^c U_B + (G_D)_{AB}\, \bar{Q}_A \varphi D_B\right] + \mathrm{h.c.} \\
\mathcal{L}_{\text{Higgs}} &= -e\left[(D_\mu\varphi)^\dagger D^\mu \varphi - \mu^2 \varphi^\dagger \varphi + \frac{\lambda}{3!}\left(\varphi^\dagger \varphi\right)^2\right] \\
\mathcal{L}_{\text{gauge}} &= -\frac{1}{2} e \left[\mathrm{Tr}\left(G_{\mu\nu} G^{\mu\nu}\right) + \mathrm{Tr}\left(W_{\mu\nu} W^{\mu\nu}\right) + \frac{1}{2} B_{\mu\nu} B^{\mu\nu}\right]
\end{aligned}$$

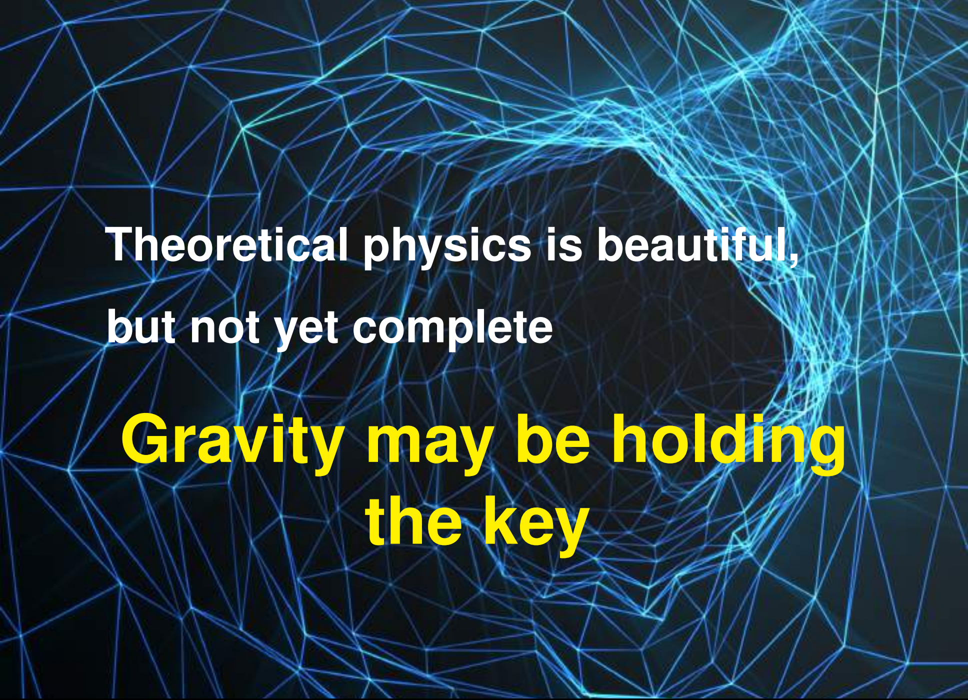
- **Spacetime of gravitational interaction**

$$S_{\text{gravity}} = \frac{1}{2\kappa} \int \mathrm{d}^4 x\, e (R - 2\Lambda + \cdots)$$

Absence of Quantum Gravity

- On one hand, we have **Quantum Field Theory** to describe the electromagnetic, strong, and weak interactions
- On the other hand, we have **General Relativity** to describe the gravity, as the dynamics of curved spacetime
- However, QFT and GR are **Not Compatible** at their face values!

[Planck & Einstein]

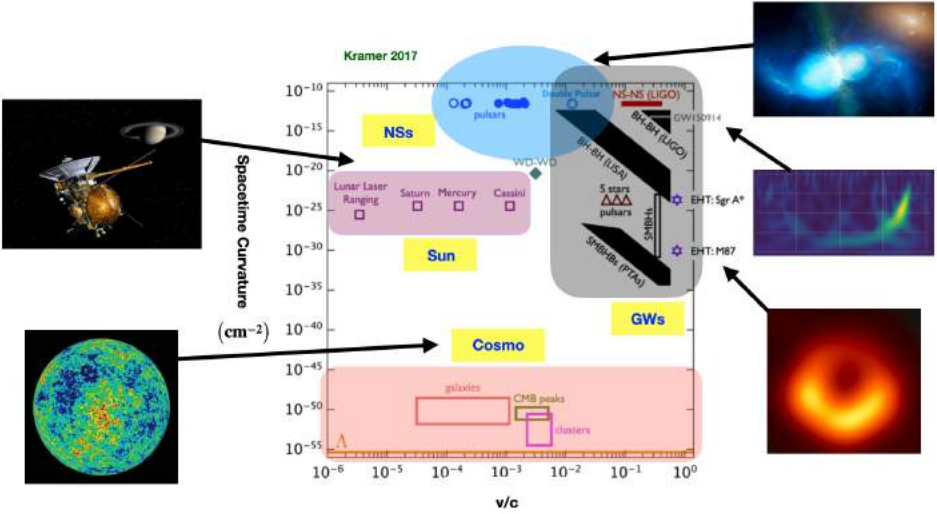


Theoretical physics is beautiful,

but not yet complete

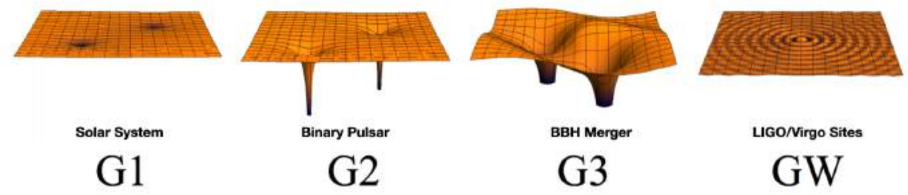
Gravity may be holding the key

Parameter Space in Gravity Tests



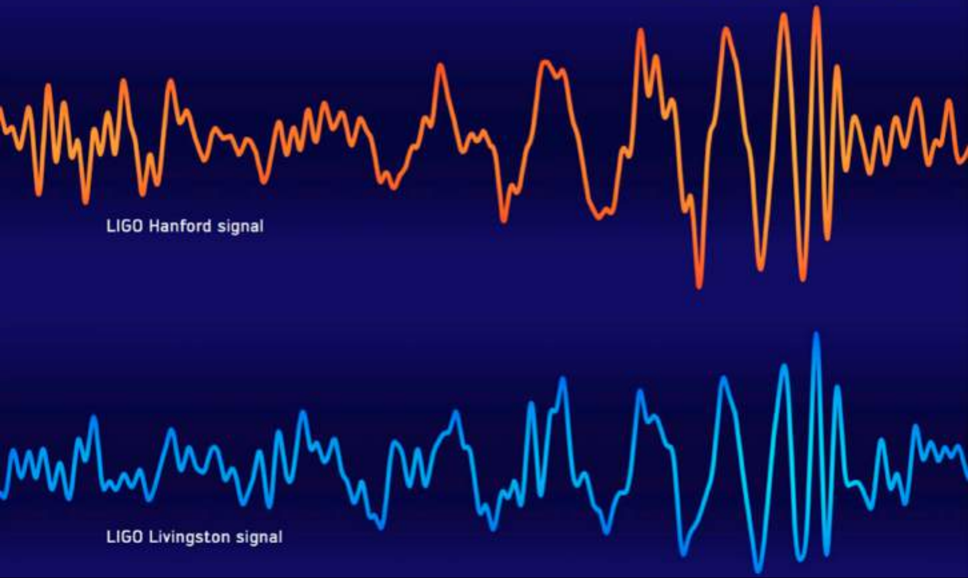
Parameter Space in Gravity Tests

- **G1**: Quasi-stationary weak-field regime
- **G2**: Quasi-stationary strong-field regime
- **G3**: Highly dynamical strong-field regime
- **GW**: Radiation regime



Wex 2014 (arXiv:1402.5594)

Gravitational-wave Data

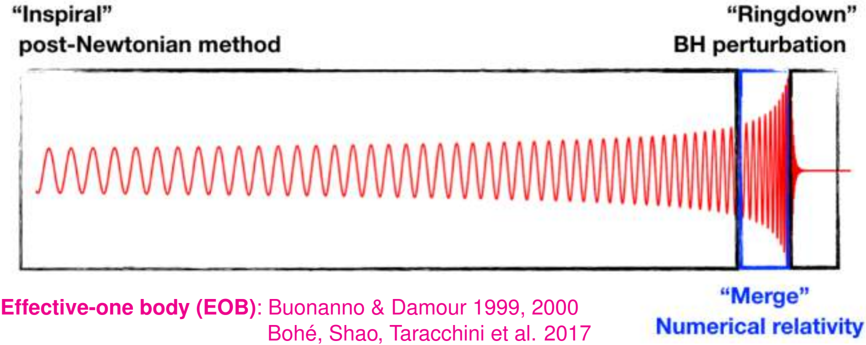


First detection!

9:50:45 UTC, 14 September 2015

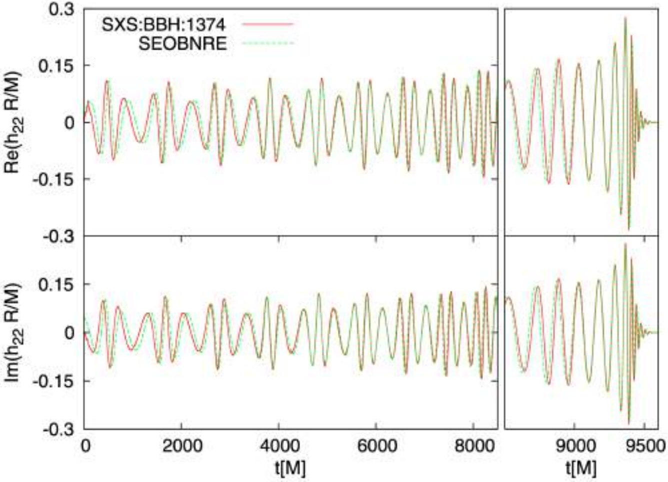
Gravitational Waveform (Time Domain)

- **Inspiral**: post-Newtonian expansion
- **Merger**: numerical relativity
- **Ringdown**: black hole perturbation



Effective-one body (EOB): Buonanno & Damour 1999, 2000
Bohé, Shao, Taracchini et al. 2017

Eccentric Waveform (Time Domain)

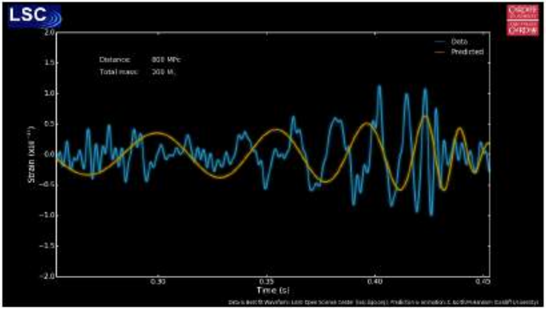


SEOBNRE: Cao & Han 2017; Liu, Cao, Shao 2020; Liu, Cao, Zhu 2021

Matched Filter

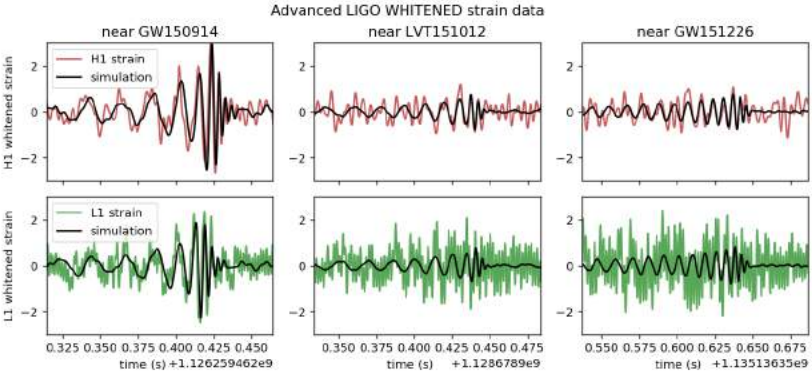
- **Matched fitlering** is a standard analysis method for **wideband** time series data [Finn 1992]

$$(\mathbf{g}|\,\mathbf{k}) \equiv 2\int_0^\infty \frac{\tilde{g}^*(f)\tilde{k}(f)+\tilde{g}(f)\tilde{k}^*(f)}{S_n(f)}\mathrm{d}f$$



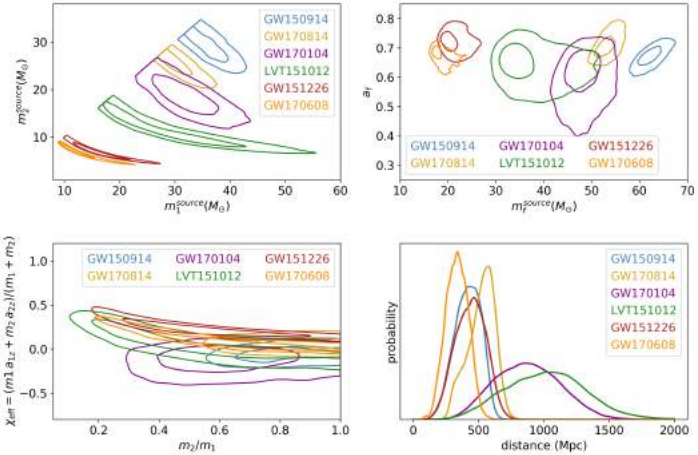
Matched Filter

- The power of **matched fitlering** lays in its ability/sensitivity to the **phase** of time-series data



Credit: Vivien Raymond / Cardiff U.

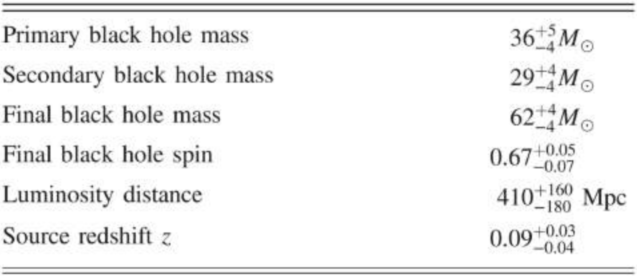
Parameter Estimation



Credit: Vivien Raymond / Cardiff U.

Parameter Estimation: GW150914

- GW data encode plenty of information of GW sources
  - Apply matched filter to data & theory

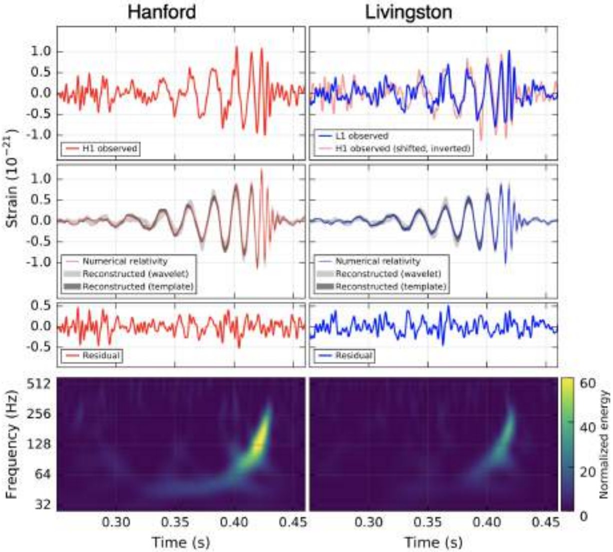


Primary black hole mass	$36^{+5}_{-4} M_\odot$
Secondary black hole mass	$29^{+4}_{-4} M_\odot$
Final black hole mass	$62^{+4}_{-4} M_\odot$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

LIGO/Virgo 2016, PRL

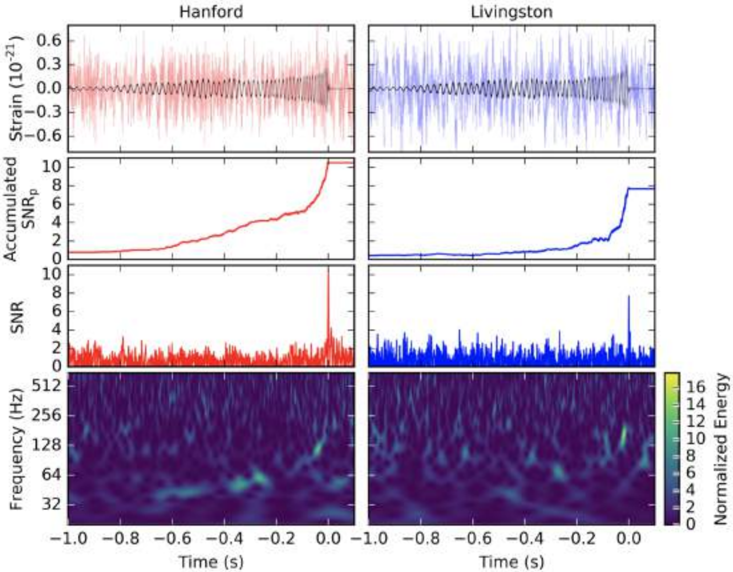
GW150914 (LIGO/Virgo 2016)

$36 + 29\,M_\odot$: 0.2 sec, SNR=23



$14 + 8\,M_\odot$: 1 sec, SNR=13

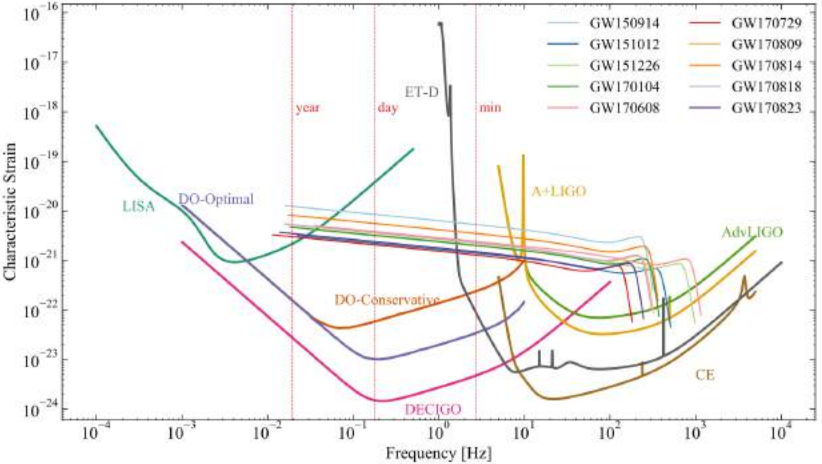
GW151226 (LIGO/Virgo 2016)



GW Transient Catalog GWTC-1 (LIGO/Virgo 2019)

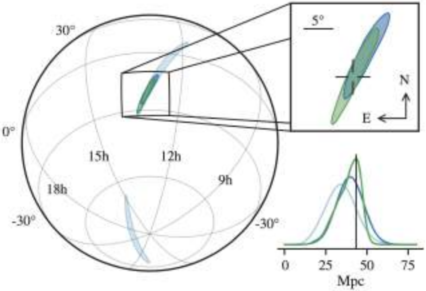
	Type	m_1 $[M_\odot]$	m_2 $[M_\odot]$	d_L $[\mathrm{Mpc}]$	Redshift z
GW150914	BBH	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$
GW151012	BBH	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	1060^{+540}_{-480}	$0.21^{+0.09}_{-0.09}$
GW151226	BBH	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	440^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$
GW170104	BBH	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	960^{+430}_{-410}	$0.19^{+0.07}_{-0.08}$
GW170608	BBH	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$
GW170729	BBH	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$
GW170809	BBH	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$
GW170814	BBH	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	580^{+160}_{-210}	$0.12^{+0.03}_{-0.04}$
GW170817	**BNS**	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	40^{+10}_{-10}	$0.01^{+0.00}_{-0.00}$
GW170818	BBH	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$
GW170823	BBH	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$

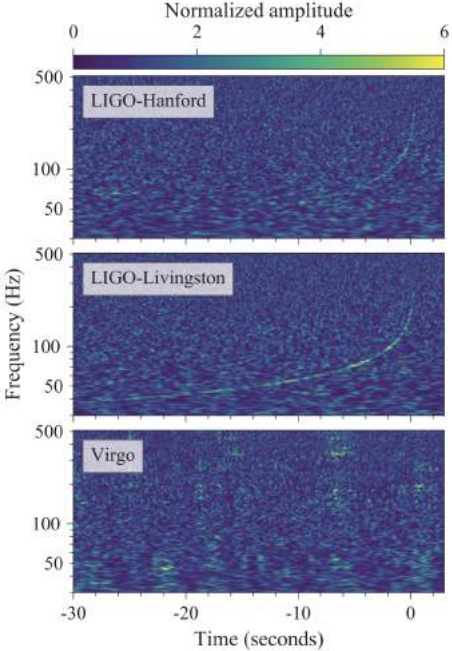
Signals of GW Events (Frequency Domain)



Liu, Shao, Zhao, Gao 2020, MNRAS [arXiv:2004.12096]

GW170817 (LIGO/Virgo 2017)

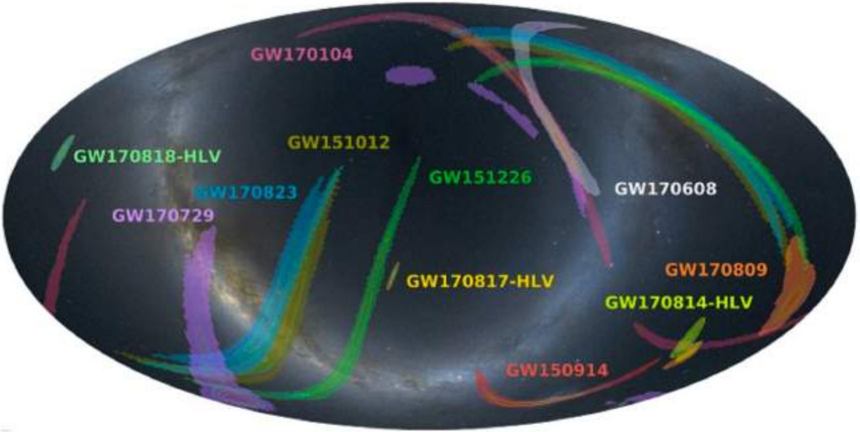




1 min, SNR=32

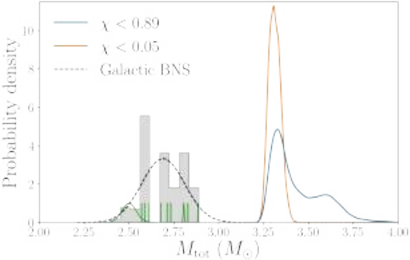
3000 cycles from 30 Hz

GWTC-1: Sky Position (LIGO/Virgo 2019)



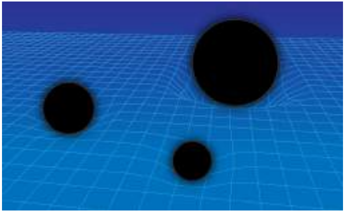
New Events from O3 (LIGO/Virgo 2020)

- **GW190412**: Observation of a Binary-Black-Hole Coalescence with Asymmetric Masses
  - $30\,M_\odot + 8\,M_\odot$; higher multipole modes
- **GW190425**: Observation of a Compact Binary Coalescence with Total Mass $\sim 3.4\,M_\odot$

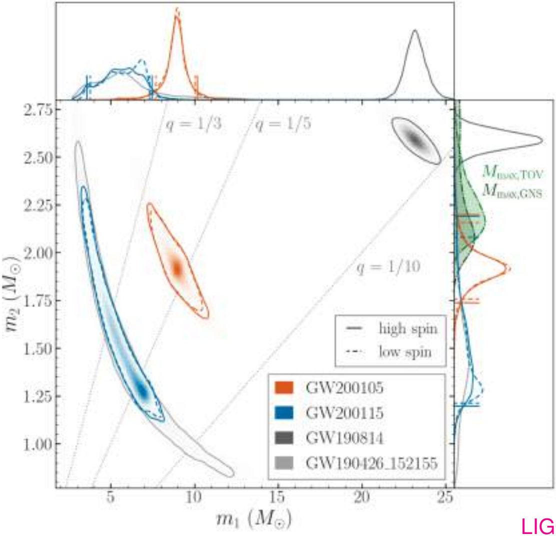


New Events from O3 (LIGO/Virgo 2020)

- **GW190521**: A Binary Black Hole Merger with a Total Mass of 150 $M_\odot$
  - $85\,M_\odot + 66\,M_\odot \Rightarrow 142\,M_\odot$
  - Intermediate mass black hole?
- **GW190814**: Gravitational Waves from the Coalescence of a 23 Solar Mass Black Hole with a 2.6 $M_\odot$ Compact Object
  - Mass gap: either the lightest black hole or the heaviest neutron star ever discovered



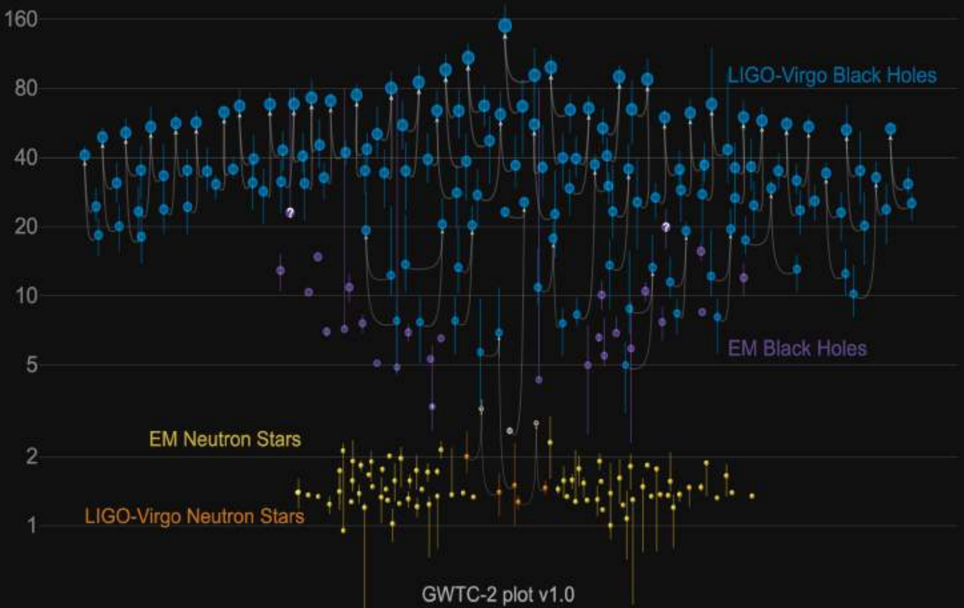
GW200105 & GW200115: BH-NS Binaries



LIGO/Virgo 2021

Masses in the Stellar Graveyard

in Solar Masses

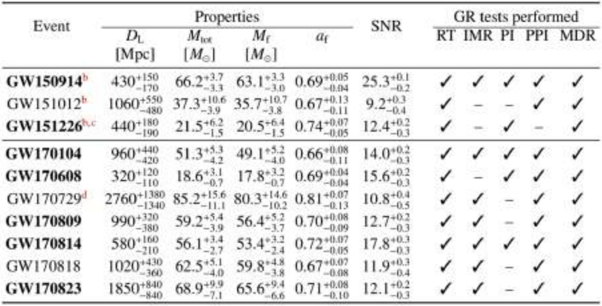


GWTC-2 plot v1.0

LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

Testing Gravity with BBHs

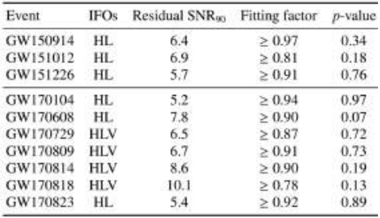
- Residual tests (RT)
- Inspiral-merger-ringdown consistency tests (IMR)
- Parameterized tests: inspiral & post-inspiral (PI & PPI)
- Modified dispersion relation (MDR)



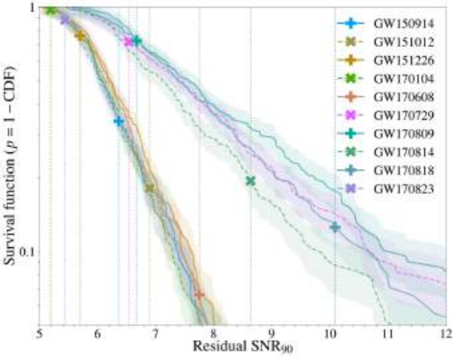
Event	Properties				SNR	GR tests performed				
	D_L [Mpc]	$M_{\rm tot}$ [$M_\odot$]	$M_{\rm f}$ [$M_\odot$]	$a_{\rm f}$		RT	IMR	PI	PPI	MDR
GW150914[b]	430^{+150}_{-170}	$66.2^{+3.7}_{-3.3}$	$63.1^{+3.3}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$25.3^{+0.1}_{-0.2}$	✓	✓	✓	✓	✓
GW151012[b]	1060^{+550}_{-480}	$37.3^{+10.6}_{-3.9}$	$35.7^{+10.7}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	$9.2^{+0.3}_{-0.4}$	✓	–	–	✓	✓
GW151226[b,c]	440^{+180}_{-190}	$21.5^{+6.2}_{-1.5}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$12.4^{+0.2}_{-0.3}$	✓	–	✓	–	✓
GW170104	960^{+440}_{-420}	$51.3^{+5.3}_{-4.2}$	$49.1^{+5.2}_{-4.0}$	$0.66^{+0.08}_{-0.11}$	$14.0^{+0.2}_{-0.3}$	✓	✓	✓	✓	✓
GW170608	320^{+120}_{-110}	$18.6^{+3.1}_{-0.7}$	$17.8^{+3.2}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$15.6^{+0.2}_{-0.3}$	✓	–	✓	✓	✓
GW170729[d]	2760^{+1380}_{-1340}	$85.2^{+15.6}_{-11.1}$	$80.3^{+14.6}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$10.8^{+0.4}_{-0.5}$	✓	✓	–	✓	✓
GW170809	990^{+320}_{-380}	$59.2^{+5.4}_{-3.9}$	$56.4^{+5.2}_{-3.7}$	$0.70^{+0.08}_{-0.09}$	$12.7^{+0.2}_{-0.3}$	✓	✓	–	✓	✓
GW170814	580^{+160}_{-210}	$56.1^{+3.4}_{-2.7}$	$53.4^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	$17.8^{+0.3}_{-0.3}$	✓	✓	✓	✓	✓
GW170818	1020^{+430}_{-360}	$62.5^{+5.1}_{-4.0}$	$59.8^{+4.8}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$11.9^{+0.3}_{-0.4}$	✓	✓	–	✓	✓
GW170823	1850^{+840}_{-840}	$68.9^{+9.9}_{-7.1}$	$65.6^{+9.4}_{-6.6}$	$0.71^{+0.08}_{-0.10}$	$12.1^{+0.2}_{-0.3}$	✓	✓	–	✓	✓

Residual Tests (LIGO/Virgo 2019)

- **Model**: best fitted model
- Residual = Data − Model
- Residual tests: consistent with noise distribution!

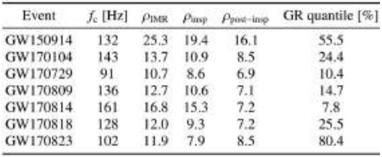


Event	IFOs	Residual SNR$_{90}$	Fitting factor	p-value
GW150914	HL	6.4	≥ 0.97	0.34
GW151012	HL	6.9	≥ 0.81	0.18
GW151226	HL	5.7	≥ 0.91	0.76
GW170104	HL	5.2	≥ 0.94	0.97
GW170608	HL	7.8	≥ 0.90	0.07
GW170729	HLV	6.5	≥ 0.87	0.72
GW170809	HLV	6.7	≥ 0.91	0.73
GW170814	HLV	8.6	≥ 0.90	0.19
GW170818	HLV	10.1	≥ 0.78	0.13
GW170823	HL	5.4	≥ 0.92	0.89

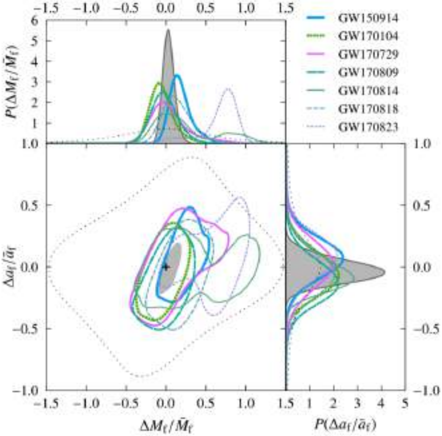


IMR Consistency Tests (LIGO/Virgo 2019)

- Parameter estimation *separately* with inpsiral and merger + ringdown
- Check consistency!



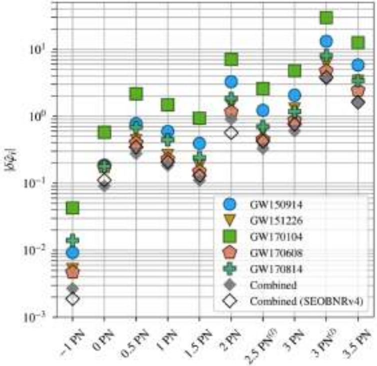
Event	f_c [Hz]	$\rho_{\rm IMR}$	$\rho_{\rm insp}$	$\rho_{\rm post-insp}$	GR quantile [%]
GW150914	132	25.3	19.4	16.1	55.5
GW170104	143	13.7	10.9	8.5	24.4
GW170729	91	10.7	8.6	6.9	10.4
GW170809	136	12.7	10.6	7.1	14.7
GW170814	161	16.8	15.3	7.2	7.8
GW170818	128	12.0	9.3	7.2	25.5
GW170823	102	11.9	7.9	8.5	80.4

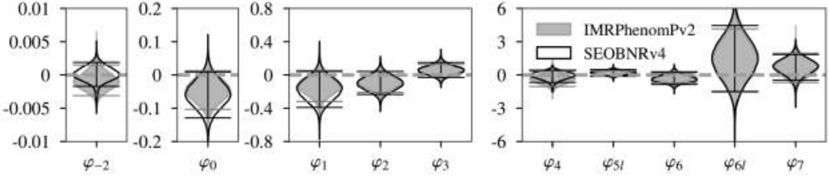


Parameterized Tests (LIGO/Virgo 2019)

$$\psi \sim \frac{3}{128\eta}(\pi f M)^{-5/3}\sum_{i=0}^{n}\varphi_i^{\rm GR}(\pi f M)^{i/3}$$

$$\varphi_i \to (1+\delta\hat{\varphi}_i)\,\varphi_i^{\rm GR}$$

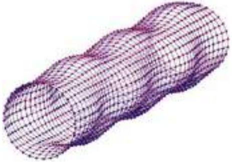




Graviton Dispersion Relation

- **GR**: massless spin-2 metric field $\Rightarrow \quad E = p$
- Lorentz-invariant massive graviton $\Rightarrow \quad E^2 = p^2 + m^2$
  - Both the phase velocity E/p and the group velocity $\partial E/\partial p$ depend on the energy/frequency of graviton
  - GWs gain *frequency-dependent* time delays when they arrive at the Earth
  - In a FRW spacetime, one has [Will 1998, PRD57:2061]

$$\Delta t_a = (1+z)\left[\Delta t_e + \frac{D}{2\lambda_g^2}\left(\frac{1}{f_e^2} - \frac{1}{f_e'^2}\right)\right]$$

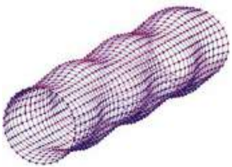


Propagation of GWs

- The extra time delay results in a phase shift in $h(f) \propto \mathrm{e}^{i\Psi(f)}$

$$\Psi(f) = \Psi_{\rm GR}(f) - \frac{\pi^2 D \mathcal{M}}{\lambda_g^2 (1+z)} (\pi \mathcal{M} f)^{-1}$$

- On the other hand, the waveform is *totally* calculable and deterministic in GR
- Therefore, GWs provide an observational window to the dispersion relation of graviton



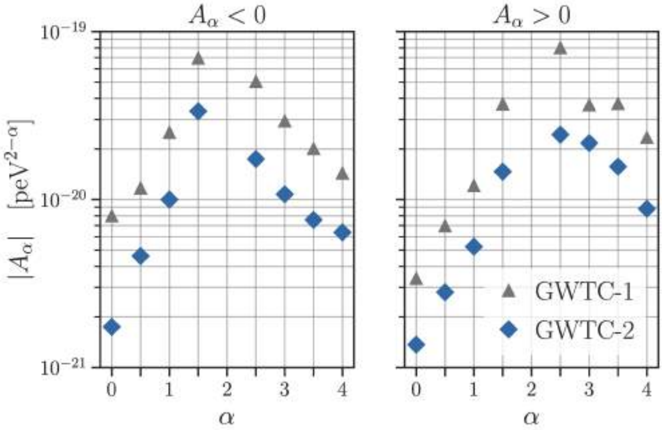
Propagation of GWs with Lorentz Violation

- Lorentz violation occurs in a few quantum gravity candidate theories [Kostelecký & Samuel 1989; Amelino-Camelia 2013]
- Dispersion relation of GWs with isotropic Lorentz violation [Mirshekari, Yunes, Will 2012]

$$\boxed{E^2 = p^2 c^2 + m_g^2 c^4 + \mathbb{A} p^\alpha c^\alpha}$$

where m_g is the graviton mass; $\mathbb{A}$ and α are two Lorentz-violating parameters

Lorentz-violating Propagation of GWs



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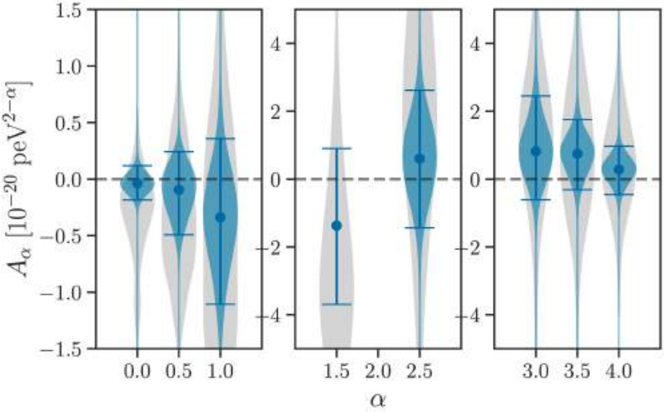
Lorentz-violating Propagation of GWs



But... such a combination is **problematic** in general

LIGO/Virgo 2021

Standard-model Extension

- The most generic linearized gravity has the Lagrangian

  [Kostelecký & Mewes 2018]

$$\boxed{\mathcal{L}_{\mathcal{K}^{(d)}} = \frac{1}{4} h_{\mu\nu} \hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} h_{\rho\sigma}}$$

where $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} = \mathcal{K}^{(d)\mu\nu\rho\sigma i_1 i_2 \cdots i_{d-2}} \partial_{i_1} \partial_{i_2} \cdots \partial_{i_{d-2}}$

- It predicts a modified dispersion relation for GWs

$$\omega = \left(1 - \zeta^0 \pm \sqrt{\left(\zeta^1\right)^2 + \left(\zeta^2\right)^2 + \left(\zeta^3\right)^2}\right) p$$

Standard-model Extension

$$\omega = \left(1 - \zeta^0 \pm \sqrt{\left(\zeta^1\right)^2 + \left(\zeta^2\right)^2 + \left(\zeta^3\right)^2}\right) p$$

$$\zeta^0 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\boldsymbol{n}}) k^{(d)}_{(I)jm}$$

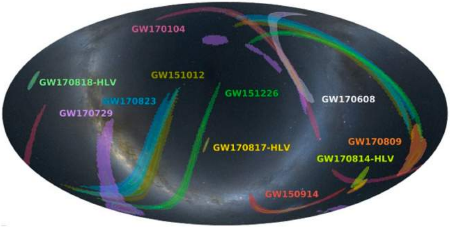
$$\zeta^1 \mp i\zeta^2 = \sum_{djm} \omega^{d-4}_{\pm 4} Y_{jm}(\hat{\boldsymbol{n}}) \left[k^{(d)}_{(E)jm} \pm i k^{(d)}_{(B)jm}\right]$$

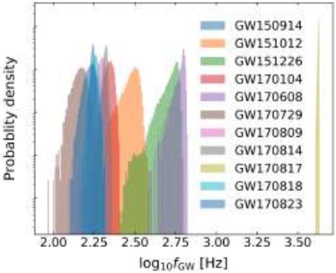
$$\zeta^3 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\boldsymbol{n}}) k^{(d)}_{(V)jm}$$

- Therefore, gravitons of different **polarization** or **frequency**, coming from different **directions** have different velocity

GWTC-1 Events

A simplified/naive approach: $|\omega_{\mathrm{GW}} \Delta t| \leq 2\pi/\rho$

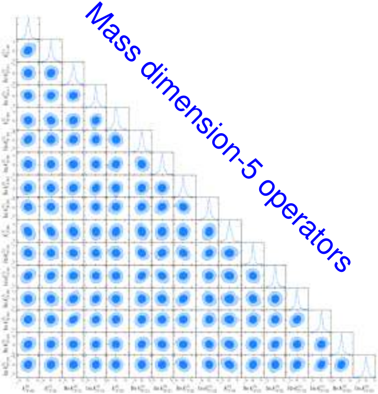




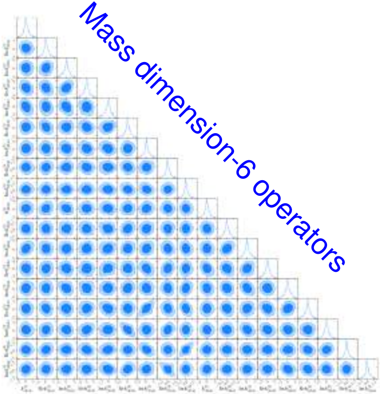
We have all the information available to perform the test

Shao 2020, PRD101:104019

Anisotropic Birefringence Combined Search



Mass dimension-5 operators



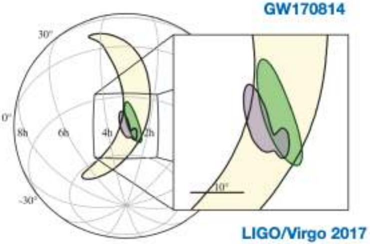
Mass dimension-6 operators

We have all the information available to perform the test

Shao 2020, PRD101:104019

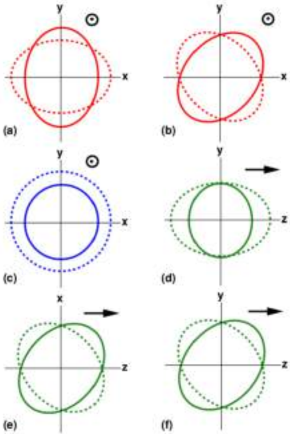
Polarization Tests (LIGO/Virgo 2019)

- **Triple** detections
  - GW170729, GW170809, GW170814, GW170818
- Bayes factors: 10^1–10^2
  - tensor vs vector
  - tensor vs scalar



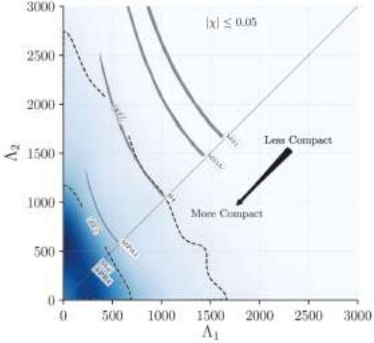
GW170814

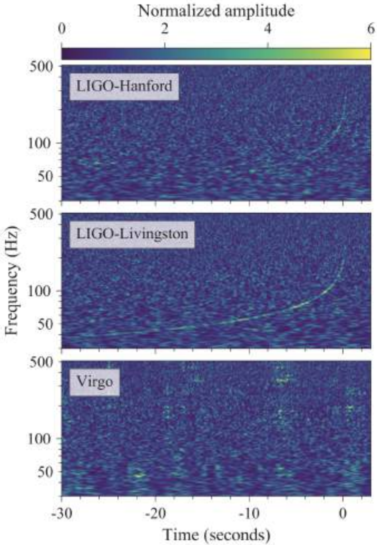
LIGO/Virgo 2017



Waveform: tidal deformability (LIGO/Virgo 2017)

- **SEOBNRv4T**
  - tidal deformability
  - equation of state



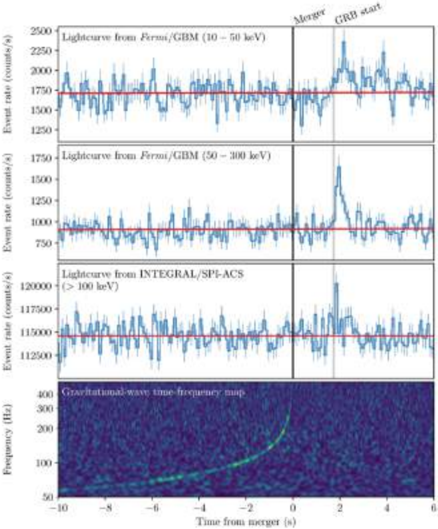


Speed of Gravity (LIGO/Virgo 2017)

- The famous 1.7 sec

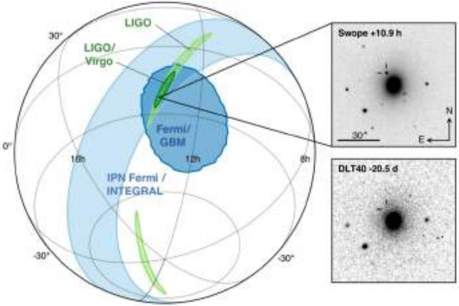
$$-3 \times 10^{-15} \leqslant \frac{\Delta v}{v_{\rm EM}} \leqslant +7 \times 10^{-16}$$

- strong implications on cosmological models
  - ... tons of PRL papers



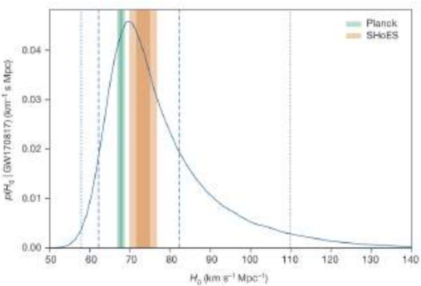
Polarization Tests (LIGO/Virgo 2019)

- Precise localization: NGC 4993
- Bayes factors
  - tensor vs vector: 10^{21}
  - tensor vs scalar: 10^{23}
- **much** tighter than BBHs

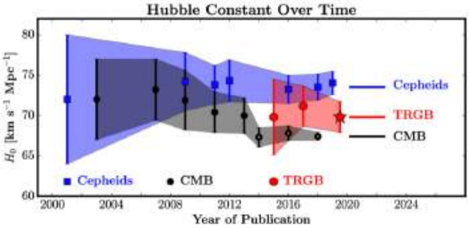


Hubble Constant (LIGO/Virgo 2017)

- By simultaneously measuring redshift and luminosity distance, GWs provide an independent way to probe cosmological parameters [Schutz 1986]



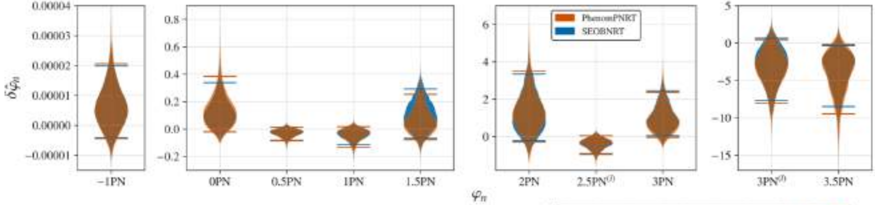
LIGO/Virgo 2017



arXiv:1907.05922 (ApJ, in press)

The Carnegie-Chicago Hubble Program

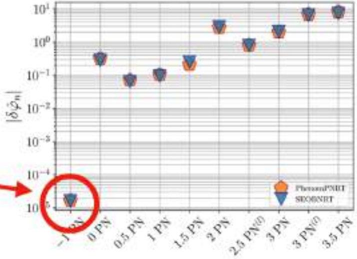
Parameterized Tests (LIGO/Virgo 2019)



$$\phi \sim \frac{3}{128\eta}(\pi f M)^{-5/3} \sum_{i=0}^{n} \varphi_i^{\rm GR} (\pi f M)^{i/3}$$

$$\varphi_i \to \left(1 + \delta\hat{\varphi}_i\right) \varphi_i^{\rm GR}$$

A tight constraint on dipole radiation



LIGO/Virgo 2019

Scalar-Tensor Gravity

$$S = \frac{c^4}{16\pi G_*} \int \frac{\mathrm{d}^4 x}{c} \sqrt{-g_*} \left[R_* - 2 g_*^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right] + S_m \left[\psi_m ; A^2(\varphi) g^*_{\mu\nu} \right]$$

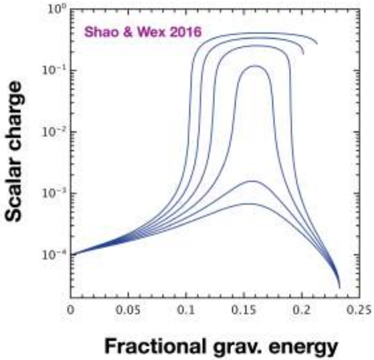
- A class of cosmologically well-motivated scalar-tensor theories, that are solely described by two theory parameters: α_0 & β_0

$$V(\varphi) = 0$$

$$A(\varphi) = \exp\left(\beta_0 \varphi^2 / 2\right) , \quad \alpha_0 = \beta_0 \varphi_0$$

Damour & Esposito-Farèse 1992; 1993; 1996

Scalar-Tensor Gravity

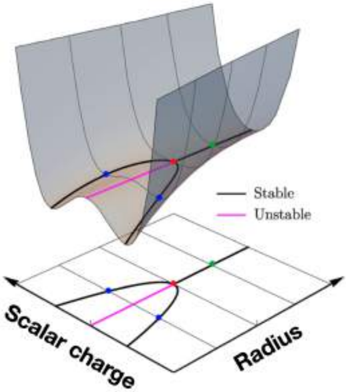


Nonperturbative **spontaneous scalarization**

could happen for isolated neutron stars

Damour & Esposito-Farèse 1992; 1993; 1996

Scalar-Tensor Gravity

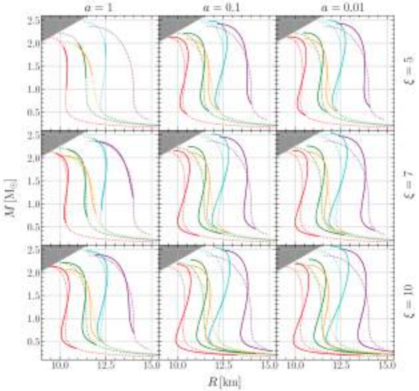


Strong-field behavior is analogous to **Landau's phase transition** after a critical point

Damour & Esposito-Farèse 1996; Esposito-Farèse 2004; Sennett, Shao, Steinhoff 2017

Massive Scalar-Tensor Gravity

- When a mass term is included, say $V(\varphi) \sim m^2\varphi^2$, a Yukawa-type suppression happens for the deviation

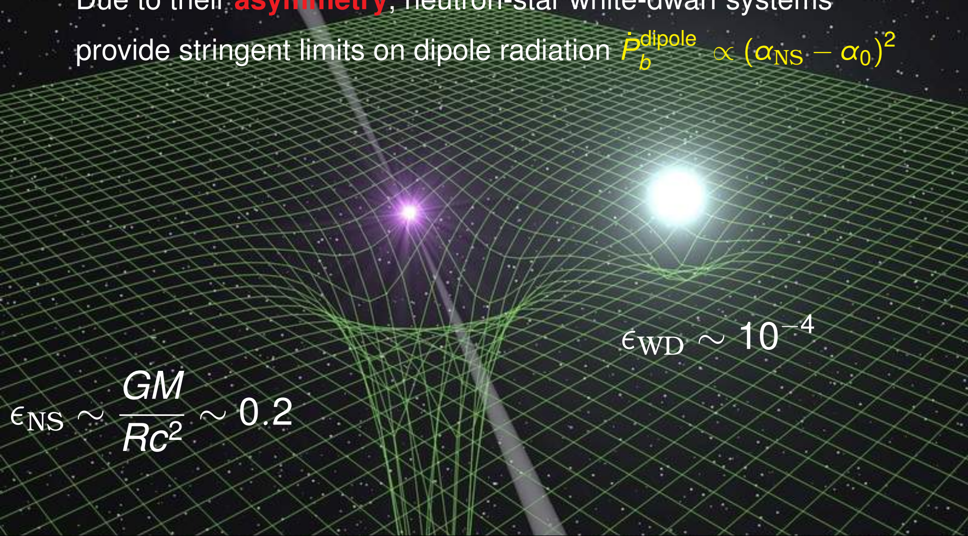


Ramazanoğlu & Pretorius 2016; Xu, Gao, Shao 2020; Hu, Gao, Xu, Shao, in prep

Strong-field gravity can be **VERY** different from weak-field gravity

Scalar-Tensor Gravity

Due to their **asymmetry**, neutron-star white-dwarf systems provide stringent limits on dipole radiation $\dot{P}_b^{\text{dipole}} \propto (\alpha_{\text{NS}} - \alpha_0)^2$

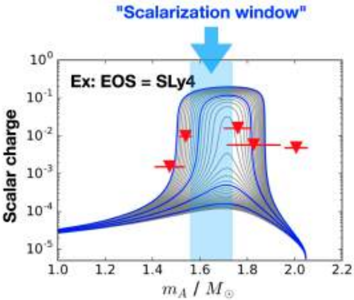


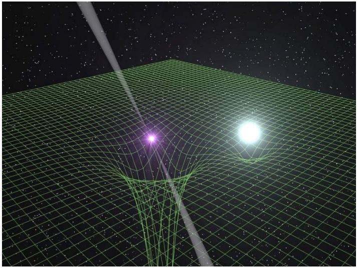
$\epsilon_{\text{WD}} \sim 10^{-4}$

$\epsilon_{\text{NS}} \sim \frac{GM}{Rc^2} \sim 0.2$

Combination of Multiple NS-WD Binaries

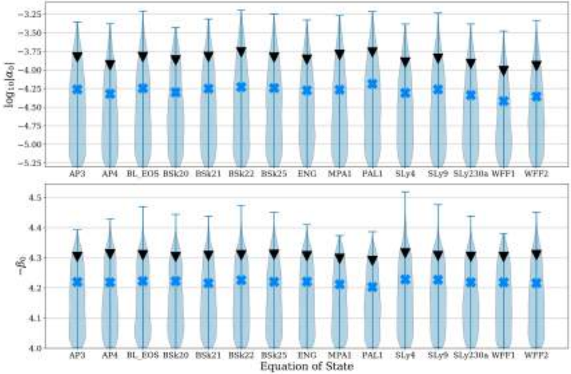
- Strong-field effects could happen at different NS masses for different EOSs [Shibata et al. 2014, PRD 89:084005]
- Combining NS-WDs put the best limits on a class of scalar tensor theories for different EOSs [Shao et al. 2017, PRX 7:041025]





Combination of Multiple NS-WD Binaries

- Reduced-order surrogate models to speed up Markov-chain Monte Carlo runs: **pySTGROM**,[2] & **pySTGROMX**[3]



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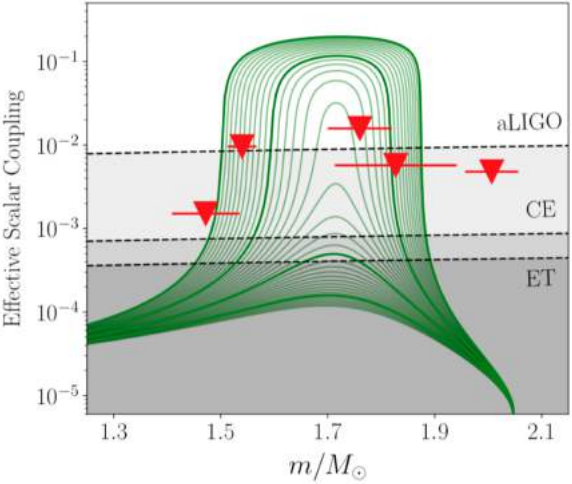
[2] https://github.com/BenjaminDbb/pySTGROM

[3] https://github.com/mh-guo/pySTGROMX

Zhao, Shao, et al. 2019

Guo, Zhao, Shao, arXiv:2106.01622

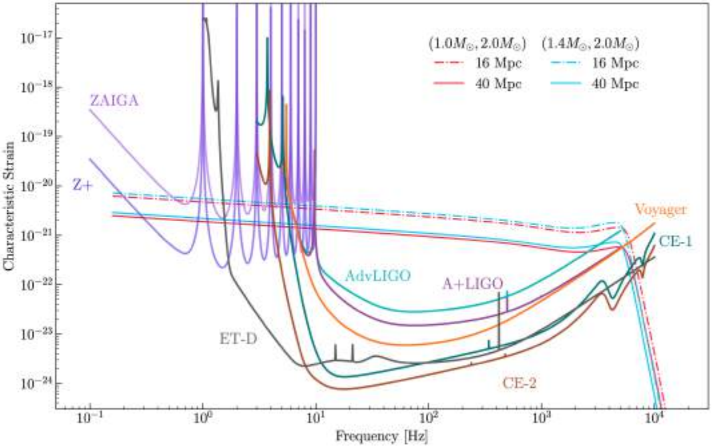
Gravitational Waves



Will 1994; Damour & Esposito-Farèse 1998; Shao et al. 2017, PRX

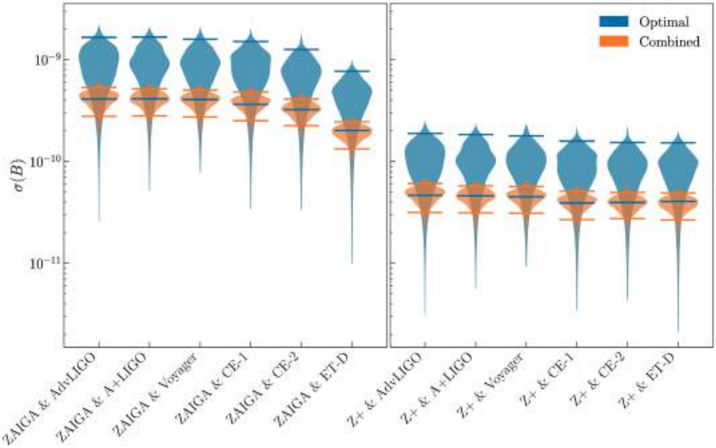
Gravitational Waves

laser interferometers & atom interferometers



Damour & Esposito-Farèse 1998; Zhao, Shao, et al., arXiv:2106.04883

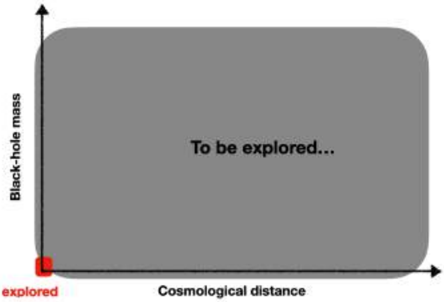
Gravitational Waves



Zhao, Shao, et al., arXiv:2106.04883

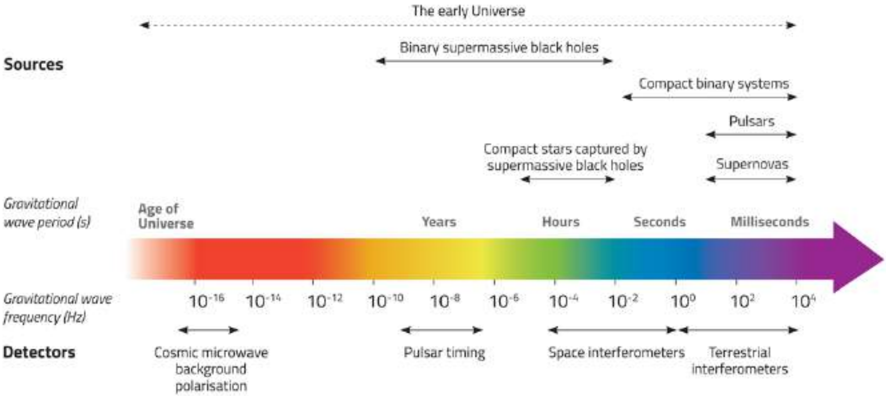
Summary

- **Einstein is still right**
- GWs launch a new era to test gravity
- Hope something new emerges *soon*



$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Albert Einstein (1915)



Only a tiny part of GW spectrum was revealed by now

Stay tuned!

An exciting era for astronomers & physicists


THANK YOU FOR LISTENING
ANY QUESTIONS?

Thank you!